Formal Mereology

How can many things be one thing? If a large part of a cat can become the whole cat (by removal of the small remaining part), why wasn't the large part the whole cat in the first place? Ancient philosophers became intrigued by such problems, but once modern logic had emerged, it was realised that a formal system for reasoning about part-whole relationships ('mereology') was possible. Standard logic could be used for the reasoning. The main system started life as the 'Calculus of Individuals', but is now called 'Classical Extensional Mereology'.

Describing the parts and the whole of something seems obvious, but a bowl of soup can be divided into parts in innumerable ways, and an account of parts must offer completeness without double-counting. You can't include the two halves and the four quarters in the same account. Hence avoiding 'overlap' is a key aim of formal mereology, and systems can be built either on overlap or on the part-whole relation, though the latter is normally preferred.

The part-whole relation seems indefinable, but indefinable concepts can be characterised using axioms, which show how they behave. A basic axiom says that **a part of a part is a part of the whol**e (so that parthood is transitive). Thus pistons are parts of engines, and also parts of cars. The other (more controversial) basic axiom allows 'unrestricted composition' (or '**fusions**'), and says that **any parts whatever can compose a whole**. Thus a pen, Mars and Lake Como can be one object. Everyday objects are just the fusions which we regularly encounter, or which interest us. The part-relation is 'asymmetric' and 'irreflexive' (as well as transitive), meaning that things can't be proper parts of one another, and nothing is a proper part of itself.

We can write '**b<<c**' to mean b is a (proper) part of c. Formally, if b and c are identical then b is a part of c, but not a '**proper part**', and in normal usage we only talk of proper parts. We write '**b o c**' to mean b overlaps c, and further symbols build up a full symbolic language for mereology. This is enough for an axiom system of mereology, but many different systems incorporate further axioms.

A further vocabulary emerges, in addition to 'overlap', 'proper part' and 'fusion' (or 'sum'). 'Distinct' or '**disjoint**' things have no parts in common, an '**atom**' is a thing with no proper parts, '**gunk**' is stuff which is unendingly divisible into parts, stuff is '**dissective**' if all the parts are the same type of stuff, and parts can be **added** or **subtracted** from an entity. Because fusions are unrestricted, there will be a single fusion of all parts, called the '**universe**', and the '**complement**' of an individual is the remaining part of the universe.

Much the most controversial aspect of classical mereology is its axiom allowing unrestricted fusions. Since metaphysics faces the awkward case of 'scattered objects' (such as the unity of a dismantled watch), the axiom avoids such difficulties by allowing unlimited scattering and unlimited difference in category for parts (so that a planet and a fruit might form one object). Some modern metaphysicians embrace what mereology offers here, and accept that unity is just a matter of convention, but traditional critics state the normal view, that a watch can cease to exist if its parts are mangled and scattered across the landscape. The mereologists typically extend their view across time, taking a 'four-dimensional' view of an object (as not wholly existing at any given moment), but again the critics are reluctant to allow a watch to be made of parts which are separated by 300 years.

The other key issue is whether a whole is more than the sum of its parts ('holism'). The nominalist view is that 'only particulars exist', and since each part is a particular, then a mereological whole adds nothing new. Mereologists need not, however, be nominalists, since they can make wholes out of abstract objects (and even Platonic Forms), and they can treat wholes as 'additions to ontology'. This seems to fit ordinary speech, where complex medium-sized physical objects are accepted as standard, and huge complexes like galaxies are happily treated as distinct things. The 'nihilist' view is that only atoms exist, and there are no fusions.

There is also an issue with the axiom saying that parthood is transitive. We say that the quarters are parts of the halves, and the halves are parts of the whole, so the quarters are also parts of the whole. However, if we simultaneously allow quarters and halves, to facilitate the transitive relation, that gives the whole six parts, which is wrong, because they overlap. There has to be some 'timeless' aspect to the picture, to make transitivity possible. It is also not clear that 'part' means quite the same thing when we talk of the bricks of a wall, and the 'parts' of a lake. We can count the bricks, but how many parts has a lake? In the face of unrestricted fusions within walls, our normal meaning for 'part' seems to collapse. Mereology works well as a formal system, but contradicts normal thinking quite dramatically.

While classical mereology is embraced by those who think of objects as parts spread over time, and who think that what counts as a whole is a mere matter of convention, for more conventional thinkers the main interest of the system is the ways in which it contrasts with set theory. In set theory the subset relation (⊂) 'includes' the members of a subset in the larger set, but the membership relation (∈) may not. Because parthood is transitive, it is inclusive, which makes it like the subsets. Set theory offers membership as a further relation. Set theory offers the slightly peculiar empty set (∅), but mereologists dislike the 'null object'.

The fact that an atomic part is rather like a singleton set {a} offers a possible link between the systems. The attraction of mereology here is that it has fewer ontological commitments, because the empty set is clearly an abstract object which can't be reduced to anything in the physical world, and yet it is the crucial foundation for standard set theory. A system based on 'parts', on the other hand, can fit in with a physicalist view of ontology (though the system might also work for abstracta, if 2 is part of 3).

In general mereology gives a helpful tool for analysis of ordinary objects, and can sort out problems with multiple overlaps, but seems a long way from common sense when it deals with parts spread across time, or parts in possible worlds. It seems to allow a possible man from one world and a possible horse from another world to form a possible centaur in a third world, or Cleopatra to form a fusion with the Eiffel Tower. Some thinkers can live with that.